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A Comparison of Results of Various Theories for Four Fundamental Constants of Physics

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Abstract

A comparison is made of theoretical values of various authors for the fine structure constant, for the proton–electron and muon–electron mass ratios, and for the gravitational constant. It is shown that a lattice ether theory developed by Aspden gives the best overall agreement with experiment.

Since the publication of Eddington's *Fundamental Theory* in 1948, several authors have made attempts to calculate fundamental constants of physics by a variety of different approaches. Accuracy of better than 100 ppm is quite common, and in several cases agreement with experiment to within 1 ppm has been obtained. Most of these theories for fundamental constants are not very well known, and the main purpose of this paper is to draw attention to the existence of some of these little-publicized works, especially those of Aspden and of Gerlovin.

Eddington (1948), using methods involving such items as "complete momentum vectors" with ten real and six imaginary components, found expressions for most of the fundamental constants, including the number of hydrogen atoms in the universe. Lenz (1951) pointed out a simple expression, $6\pi^5$, which gave the proton–electron mass ratio quite well. Good (1970) gave this expression geometrical meaning in terms of Eddington's ideas, and used an empirical modification of these to find new formulas for the neutron–proton mass difference and for the gravitational constant. Wyler (1969, 1971) obtained a formula for the fine structure constant by using sophisticated arguments involving special geometries, and also found Lenz and Good's expression for the proton–

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electron mass ratio by an extension of these arguments. Aspden (1969) used a lattice ether theory to obtain many of the fundamental constants of physics to within 0.1% accuracy. More recently, in collaboration with the author, he developed his ideas to obtain more accurate results for the fine structure constant (Aspden and Eagles, 1972) for the proton–electron mass ratio (Aspden, 1975; Aspden and Eagles, 1975), for the gravitational constant (Aspden, 1975), and for the muon–electron mass ratio (Aspden, to be published). By use of a stochastical approach to quantum theory involving interaction of charges with a fluctuating zero-point field, Surdin (1971) obtained a very approximate expression for the fine structure constant, and found a relation between the gravitational constant and the radius of the universe. Jehle (1971, 1975) has also given very approximate numerical calculations of the fine structure constant by use of an unorthodox approach to elementary particle theory based on distributions of loops of quantized magnetic flux. Ross (1972) developed a model for the muon involving an electron orbited by a massless spin-1 wave, and deduced a simple formula for the muon mass. Tennakone (1974) obtained the same formula using a different model. Gerlovin (1971, 1973, 1974) has developed a comprehensive thoery for calculating most fundamental constants, including the fine structure constant, the proton–electron and muon–electron mass ratios, and the gravitational constant. His theory involves a model for particles according to which they contain two sets of charges moving in circular orbits with different radii about a common center with relativistic velocities. The condition that the charge system does not radiate imposes relations between the parameters characterizing the sets of charges and orbits, and so restricts the possible values of these parameters. Stability conditions limit these values still further. Particles fall into a number of series, and the proton and electron are the most stable particles of the first and third series, respectively. Lewis (1973) suggested a formula for the fine structure constant in terms of the proton, neutron, and electron masses, based on a theory of a "proton–electron–antineutrino oscillator." Although numerical formulas for dimensionless constants do not appear in two long papers by MacGregor (1974a, b), they are probably worth mentioning in this context because of many suggested relationships between particle masses and those of postulated basic light quarks of masses corresponding to energies of 70 and 330 MeV. The 70 MeV quantum also plays an important part in the work of Lewis. Delaney (1974) developed a semiclassical model for elementary particles involving three types of quarks moving in circular orbits about a common center. He obtained two expressions for the proton–electron mass ratio, one being the $6\pi^5$ value mentioned before, with the other depending on the fine structure constant α. By equating the two expressions he found a value for α. Pradhan and Khare (1974) suggested a first approximation to the fine structure constant as a result of a study of equal time commutators in a field theory with a fundamental length, while Lord et al. (1974) used a theory of strong gravity to obtain a rough estimate for the proton–electron mass ratio. Alexanian (1975) suggested an approximate relationship between m_e, m_p,

m_π, and α based on a theory for the early history of the universe. Lastly, an accurate formula for α^{-1} has been proposed by Mellen (1975), with a partial theory based on the requirement that there should be an odd integral number of Compton wavelengths in a hydrogen Bohr orbit with a spiral twist added.

Some useful comments on Eddington's work are contained in a book by Slater (1957). An extended discussion of Wyler's rather terse work is given by Robertson (1971), and a more physical derivation of his formula for α^{-1} has been given by Vigier (1973).

Except for the very approximate results in the theories of Surdin, Jehle, Pradhan and Khare, Lord et al., and Alexanian, the values obtained by the various authors mentioned for α^{-1}, m_p/m_e, m_μ/m_e, and G are shown in Table 1. Formulas are given except when too complicated to be included conveniently in the table. The currently accepted experimental results are also shown. These are taken from Cohen and Taylor (1973). We write M_H for the mass of a hydrogen atom and use the notation $\beta = (137/136)$ in Eddington's formulas. Nearly all the theories mentioned introduce a large number of arbitrary assumptions. However, in most cases qualitative assumptions lead to quantitative results.

It can be seen from the table that Aspden's theory gives the best overall agreement with experiment. For the fine structure constant and for the proton-electron mass ratio his results lie within 0.91 and 0.44 ppm of the most probable experimental values, only just outside the standard deviation uncertainty limits of 0.82 and 0.38 ppm quoted by Cohen and Taylor. For the muon-electron mass ratio and for the gravitational constant Aspden's results lie well within the experimental uncertainty ranges of 2.3 and 615 ppm, respectively. The only theoretical results which come closer to experiment than any of Aspden's are those of Wyler and of Mellen for α^{-1}. The related expression of Wyler for the proton-electron mass ratio is in error by about 18 ppm.

At first sight it might seem surprising that so many different types of formula give results in fair agreement with experiment. However, this becomes more understandable when it is realized that Roskies (1971) reported four other numbers besides Wyler's involving products of simple fractional powers of 2, 3, 5, and π, which gave agreement with the experimental value of α^{-1} to within 1 ppm. Also Peres (1971) has given arguments to show why it would be surprising if it were not possible to find integers x, y, z, and t such that $(2^x 3^y 5^z \pi^t)^{1/4}$ lies within a few parts per million of any particular number aimed at, such as the experimental result for α^{-1}. It might be argued that this type of result implies that none of the theories discussed represent more than plausibility arguments built around numerical formulas found to give agreement with experiment. However, some of the theories show little evidence of having been consciously developed in this way.

A summary of the parts of Aspden's work that are relevant to the calculation of values of the four fundamental constants considered, together with a brief discussion of some difficulties that arise in connection with his ideas, is being submitted for publication elsewhere.

TABLE 1. Theoretical results of various authors for α^{-1}, m_p/m_e, m_μ/m_e, and G in comparison with experimental values (Cohen and Taylor 1973)

Constant	Formula	Numerical value	Authors
α^{-1}	$137\beta^{1/24}$	137.042	Eddington, 1948[a]
	$(2^4 5!/\pi^5)^{1/4}(8\pi^4/9)$	137.036082	Wyler, 1969
	$108\pi(8/1843)^{1/6}$	137.035915	Aspden and Eagles, 1972
		137.0366	Gerlovin, 1973, p. 137
	$(2m_n - m_p)[18.5m_e - 2(m_n - m_p)]^{-1}$	137.019±0.003	Lewis, 1973[b]
	$4\pi^5/9 + 37/36$	137.03653	Delaney, 1974
	$(137^2 + \pi^2)^{1/2}$	137.036016	Mellen, 1975
		137.03604 (11)	Experiment
m_p/m_e	$0.1\beta^{-5/6}[68 + (68^2 - 10\beta^{5/6})^{1/2}]^2$	1836.34	Eddington, 1948
	$6\pi^5$	1836.118	Lenz, 1951; Good, 1970; Wyler, 1971; Delaney, 1974
		1836.10	Gerlovin, 1973, p. 138
	$(\alpha^{-1} - 37/36)(27/2)$	1836.11	Delaney, 1974
	$(3/4)108^3\pi^2(1843)^{-4/3}[(3/2)^{1/2} - 1]^{-1}$	1836.15232	Aspden, 1975, p. 72; Aspden and Eagles, 1975

		1836.15152 (70)	Experiment
m_μ/m_e		206.308	Gerlovin, 1973, p. 138
	$1 + (3/2)\alpha^{-1}$	206.554	Ross, 1972; Tennakone, 1974
		206.76847	Aspden (to be published)
		206.76865 (47)	Experiment
G	$\beta^{1/6}(137\pi/10)(136)^{1/2}(3/10)^{1/2}2^{-128}(e/M_H)^2$	$6.6631 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$	Eddington, 1948[c]
	$\alpha^{-1}(3 \times 136 \times 2^{256} \times 6 \times 32640 \times \pi^{68}/68!)^{-2} \times (1 + 0.1\alpha)^{-2}(e/m_e)^2$	$6.6731 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$	Good, 1970
	$1.000888 \times (9/32\pi^2)(\lambda_p R_\infty)^4(e/m_p)^2$	$6.6725 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$	Gerlovin, 1973, 1974; Krat and Gerlovin, 1974[d]
	$(1+\epsilon)^2[4\pi/(108\pi)^3 g^4]^2(e/m_e)^2$, where $g = (9/4)[(3/2)^{1/2}(m_p/m_e) + 1] + \frac{1}{2} = 5062.59$ and $\epsilon = (4\pi/3)(1843)(108\pi)^{-3} = 1.98 \times 10^{-4}$	$6.6718 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$	Aspden, 1975, p. 72
		$6.6720(41) \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$	Experiment

[a] The power of β in Eddington's theory could perhaps depend on the method of measurement. We have noted Eddington's opinion on p. 64 of his book (1953 ed.) that determinations of e/h involve the "true" charge and Planck's constant, while the other charge in the expression for α will be the "spectroscopically controlled" charge $e' = e\beta^{-1/24}$.

[b] To obtain a numerical value and estimated error from the formula of Lewis we used values for the proton and neutron masses and mass difference given by the Particle Data Group (1974).

[c] If we substitute m_p for M_H in Eddington's expression for G, we calculate $G = 6.6704 \times 10^{-11}\,\mathrm{Nm^2kg^{-2}}$.

[d] This numerical value for G differs slightly from that found by Gerlovin because we used more recent data in his formula.

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